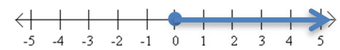
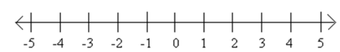
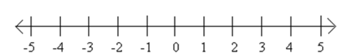
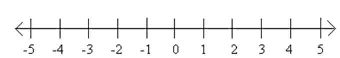
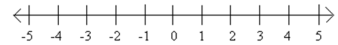
Project 11: More detailed practice on topics for MAT 56

We can use the following properties to rewrite equations (to put them in a particular form like $y - mx + b$, or to solve them, or to combine multiple equations into a single equation).

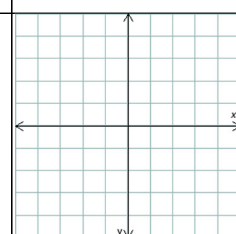
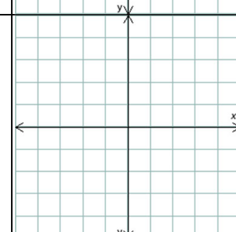
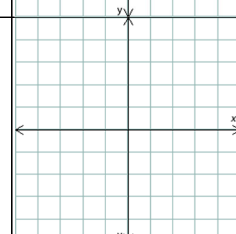
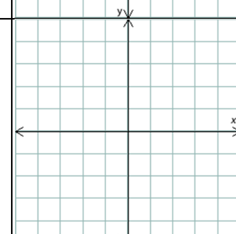
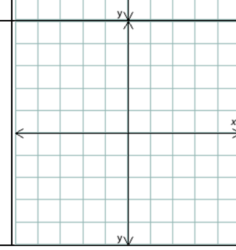
Properties that apply to equations specifically:
$a = b \leftrightarrow b = a$
$a = b \leftrightarrow a + c = b + c$ $a < b \leftrightarrow a + c < b + c$
$a = b \leftrightarrow a \cdot c = b \cdot c \quad (whenever\ c \neq 0)$ $a < b \leftrightarrow \begin{cases} a \cdot c < b \cdot c & if\ c > 0 \\ a \cdot c > b \cdot c & if\ c < 0 \end{cases}$
$a = b \leftrightarrow \frac{a}{c} = \frac{b}{c} \quad (whenever\ c \neq 0)$ $a < b \leftrightarrow \begin{cases} \frac{a}{c} < \frac{b}{c} & if\ c > 0 \\ \frac{a}{c} > \frac{b}{c} & if\ c < 0 \end{cases}$
$a = b\ and\ c = d \leftrightarrow a + c = b + d$ $a = b\ and\ c = d \leftrightarrow na + mc = nb + md$ $for\ any\ numbers\ n\ and\ m$
$ab = 0 \leftrightarrow a = 0\ or\ b = 0$
$ax^2 + bx + c = 0 \leftrightarrow ax^2 + b_1x + b_2x + c = 0 \quad where$ $* b_1, b_2\ are\ whole\ numbers$ $* b_1 \cdot b_2 = ac$ $* b_1 + b_2 = b$
Properties for expressions, that can also be used on expressions inside equations:
$a(b + c) = ab + ac$ $(a + b)c = ac + bc$ $a(x_1 + \cdots + x_n) = ax_1 + \cdots + ax_n$ $(x_1 + \cdots + x_n)c = x_1c + \cdots + x_nc$
$x^n = \underbrace{x \cdot \ldots \cdot x}_{n-many\ times}$
$x^{-n} = \frac{1}{x^n} \quad (whenever\ x \neq 0)$
$\frac{x}{1} = x$
$\frac{x}{x} = 1 \quad (whenever\ x \neq 0)$
$\frac{ab}{cd} = \frac{a}{c} \cdot \frac{b}{d} \quad (whenever\ c, d \neq 0)$ $\frac{x_1 \cdot \ldots \cdot x_n}{y_1 \cdot \ldots \cdot y_n} = \frac{x_1}{y_1} \cdot \ldots \cdot \frac{x_n}{y_n} \quad (whenever\ y_1, \ldots, y_n \neq 0)$
$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad (whenever\ c \neq 0)$ $\frac{x_1 + \cdots + x_n}{y} = \frac{x_1}{y} + \cdots + \frac{x_n}{y} \quad (whenever\ y \neq 0)$
$\sqrt{x^2} = x \quad (whenever\ x \geq 0)$
$\sqrt{ab} = \sqrt{a}\sqrt{b}$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad (whenver\ b \neq 0)$

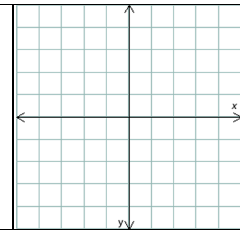
Just like with equations, use the properties on the first page of this project to solve each of the following inequalities for the given variable, and check your answer by testing two sample values. Then graph the solution set.

Solve for x:	Check answer by testing two values:	Graph the solution set
Example: $10 - x \leq 2(x+5)$ $[10 + -x] + -10 \leq [2(x+5)] + -10$ $10 + -x + -10 \leq 2(x+5) + -10$ $10 + -x + -10 \leq 2 \cdot x + 2 \cdot 5 + -10$ $10 + -x + -10 \leq 2x + 10 + -10$ $-x + 10 + -10 \leq 2x + (10 + -10)$ $-x + (10 + -10) \leq 2x + 0$ $-x + 0 \leq 2x$ $-x \leq 2x$ $-x + -2x \leq 2x + -2x$ $-3x \leq 0$ $\frac{-3x}{-3} \geq \frac{0}{-3}$ $1x \geq 0$ $x \geq 0$	Testing $x = -1$, which should be false, because $-1 \ngeq 0$: $10 - (-1) \leq 2((-1) + 5)$ $10 + 1 \leq 2(4)$ $11 \leq 8$ → FALSE! ✓ Testing $x = 1$, which should be true, because $1 \geq 0$: $10 - (1) \leq 2((1) + 5)$ $9 \leq 2(6)$ $9 \leq 12$ → TRUE! ✓	
1. $3 - 2x \leq 9 - 5x$		
2. $5 - x > 3 - 3x$		
3. $-2x + 4 < 3 - 4(x+2)$		

4. $12 - 2(3x - 1) < 5(2 - x)$		

For each of the pairs of equations, first put <u>each</u> equation into point-slope form ($y = mx + b$), then identify the slope and the intercept of <u>each</u> equation in the pair, and then graph the pair of lines. Finally, identify the point where the pair of lines intersect, or state that they are parallel or the same line:

Equations of the lines:	**Slope intercept forms:**	**Slope positive, negative, zero, or undefined?**	**Slope has magnitude less than 1 or greater than 1?**	**Slope:**	***y*-int:**	**Graph**	**Point of intersection: Or if no intersection, parallel or same line?**
5. $y = -x + 1$ $x = 5 - 3y$							
6. $2x + 3y = 3$ $y = -\frac{2}{3}x + 1$							
7. $3x - y = 6$ $2x + y = 4$							
8. $x - 4y = 4$ $3x + 2y = -2$							
9. $2x - 3y = 9$ $y = \frac{2}{3}x - 3$							

10. $2x - 3y = 6$ $3x + 4y = -8$							

Solving Systems of Linear Equations: For each of the pairs of equations, pick a property that will allow you to combine the two equations into a single equation with only one variable. Then solve for that variable. After solving for the first variable, substitute that value into either original equation to find the value of the second value. Then check your answer.

Properties: 1) $a = b \text{ and } c = d \leftrightarrow a + c = b + d$ 2) $a = b \text{ and } c = d \leftrightarrow na + mc = nb + md$ $\text{for any numbers } n \text{ and } m$

Equations of the lines:	**Which property is likely best to merge these two equations into <u>a single equation with only one variable</u>?** **Substitution or one of the properties above?**	**Use that property here to combine the two equations into <u>a single equation with only one variable:</u>** For substitution, which equation should be solved for which variable? If using the property #2 above, what should n and m be?	**Solve to get a number value for the first variable:**	**Solve to get a number value for the second variable:**	**Rewrite final solution here:**	**Check that solution by substituting <u>both</u> the *x* and *y* values into <u>both</u> equations:**
11. $y = -x + 1$ $x = 5 - 3y$						
12. $2x + 3y = 3$ $y = -\frac{2}{3}x + 1$						
13. $3x - y = 6$ $2x + y = 4$						
14. $x - 4y = 4$ $3x + 2y = -2$						
15. $2x - 3y = 9$ $y = \frac{2}{3}x - 3$						

16. $2x - 3y = 6$ $3x + 4y = -8$						

Factoring: Factoring is just the reverse of multiplying!

Examples:

Original Product—Multiply out	**Write process in reverse**	**Describe pattern**
$2x(x-3) = 2x(x+-3)$ $= 2x \cdot x + 2x \cdot -3$ $= 2x^2 + 2 \cdot -3 \cdot x$ $= 2x^2 + -6x$ or $2x^2 - 6x$	$2x^2 - 6x = 2x^2 + -6x$ $= 2x \cdot x + 2x \cdot -3$ $= 2x(x+-3)$	To factor this expression, we need to look for something that we can factor out of each term (the greatest common factor).
$(2x+1)(3y-2) = (2x+1)(3y+-2)$ $= (2x)(3y+-2) + 1(3y+-2)$ $= 2x \cdot 3y + 2x \cdot -2 + 1 \cdot 3y + 1 \cdot -2$ $= 2 \cdot 3 \cdot x \cdot y + 2 \cdot -2 \cdot x + 3y + -2$ $= 6xy + -4x + 3y + -2$ or $= 6xy - 4x + 3y - 2$	$6xy - 4x + 3y - 2$ $= 6xy + -4x + 3y + -2$ $= (6xy + -4x) + (3y + -2)$ $= 2x(3y+-2) + 1(3y+-2)$ $= (2x+1)(3y+-2)$ or $= (2x+1)(3y-2)$	To factor this expression, we need to: • group the first two terms and the last two terms together; and • see that each of these groups has the form of the right side of the identity $(a+b)c = ac + bc$.
$5ab(2a+b)(3b-1)$ $= 5ab(2a+b)(3b+-1)$ $= 5ab[(2a)(3b+-1) + b(3b+-1)]$ $= 5ab(2a \cdot 3b + 2a \cdot -1$ $+b \cdot 3b + b \cdot -1)$ $= 5ab(2 \cdot 3 \cdot a \cdot b + 2 \cdot -1 \cdot a$ $+3 \cdot b \cdot b + -1b)$ $= 5ab(6ab + -2a + 3b^2 + -1b)$ $= 5ab \cdot 6ab + 5ab \cdot -2a$ $+5ab \cdot 3b^2 + 5ab \cdot -1b$ $= 5 \cdot 6 \cdot a \cdot a \cdot b \cdot b + 5 \cdot -2 \cdot a \cdot a \cdot b$ $+5 \cdot 3 \cdot a \cdot b \cdot b^2 + 5 \cdot -1 \cdot a \cdot b \cdot b$ $= 30a^2b^2 + -10a^2b + 15a \cdot b \cdot b \cdot b$ $+ - 5ab^2$ $= 30a^2b^2 + -10a^2b + 15ab^3 + -5ab^2$ or $= 30a^2b^2 - 10a^2b + 15ab^3 - 5ab^2$	$30a^2b^2 - 10a^2b + 15ab^3 - 5ab^2$ $= 30a^2b^2 + -10a^2b + 15ab^3$ $+ - 5ab^2$ $= 5ab \cdot 6ab + 5ab \cdot -2a$ $+5ab \cdot 3b^2 + 5ab \cdot -1b$ $= 5ab(6ab + -2a + 3b^2 + -1b)$ $= 5ab([6ab + -2a]$ $+[3b^2 + -1b])$ $= 5ab(2a[3b + -1] + b[3b + -1])$ $= 5ab([2a+b][3b+-1])$ $= 5ab(2a+b)(3b+-1)$ or $= 5ab(2a+b)(3b-1)$	**To factor this expression, we first need to see that the whole expression has the form of the right side of the identity $(x_1 + \cdots + x_n)c = x_1c + \cdots x_nc$.** Then we need to: • group the first two terms and the last two terms together; and • see that each of these groups has the form of the right side of the identity $(a+b)c = ac + bc$.
$(5x-3)(2x+5) = (5x+-3)(2x+5)$ $= (5x)(2x+5) + -3(2x+5)$ $= 5x \cdot 2x + 5x \cdot 5 + -3 \cdot 2x + -3 \cdot 5$ $= 5 \cdot 2 \cdot x \cdot x + 5 \cdot 5 \cdot x + -6x + -15$ $= 10x^2 + 25x + -6x + -15$ $= 10x^2 + 19x + -15$ or $= 10x^2 + 19x - 15$	$10x^2 + 19x - 15$ $= 10x^2 + 19x + -15$ $= 10x^2 + 25x + -6x + -15$ $= (10x^2 + 25x) + (-6x + -15)$ $= 5x(2x+5) + -3(2x+5)$ $= (5x+-3)(2x+5)$ or $= (5x-3)(2x+5)$	To factor this expression, we need to: • **first break up the middle term into two separate terms** that will allow us to get two groups, each with the form of the right side of the $(a+b)c = ac + bc$; • then group the first two terms and the last two terms together; and • then see that each of these groups has the form of the right side of the identity $(a+b)c = ac + bc$
$2x^2(x-7)(3x+2)$ $= 2x^2(x+-7)(3x+2)$ $= 2x^2[x(3x+2) + -7(3x+2)]$ $= 2x^2(x \cdot 3x + x \cdot 2 + -7 \cdot 3x + -7 \cdot 2)$ $= 2x^2(3x^2 + 2x + -21x + -14)$ $= 2x^2(3x^2 + -19x + -14)$ $= 2x^2 \cdot 3x^2 + 2x^2 \cdot -19x + 2x^2 \cdot -14$ $= 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x + 2 \cdot -19 \cdot x \cdot x \cdot x$ $+2 \cdot -14x^2$ $= 6x^4 + -38x^3 + -28x^2$ or $= 6x^4 + -38x^3 - 28x^2$	$6x^4 + -38x^3 - 28x^2$ $= 6x^4 + -38x^3 + -28x^2$ $= 2x^2(3x^2 + -19x + -14)$ $= 2x^2(3x^2 + 2x + -21x + -14)$ $= 2x^2([3x^2 + 2x] + [-21x + -14])$ $= 2x^2(x[3x+2] + -7[3x+2])$ $= 2x^2([x+-7][3x+2])$ $= 2x^2(x+-7)(3x+2)$ or $= 2x^2(x-7)(3x+2)$	To factor this expression, we first need to see that the whole expression has the form of the right side of the identity $(x_1 + \cdots + x_n)c = x_1c + \cdots x_nc$. Then we need to: • first break up the middle term into two separate terms that will allow us to do further factoring; and • then group the first two terms and the last two terms together; and • then see that each of these groups has the form of the right side of the identity $(a+b)c = ac + bc$

Factoring trinomials: For most of the patterns we observed, it is clear after working out several examples how we would reverse the procedure. However, there is one exception: In some cases when we multiply two binomials, two of the terms that result are like terms and are therefore combined, resulting in three terms instead of four. Before these trinomials (three terms) can be factored, we have to find a way to break up the middle term. Let's look at the pattern:

$$
\begin{aligned}
&(2x+3)(4x+5)\\
&= 2x\cdot 4x + 2x\cdot 5 + 3\cdot 4x + 3\cdot 5\\
&= 2\cdot 4x^2 + 2\cdot 5x + 3\cdot 4x + 3\cdot 5\\
&= 2\cdot 4x^2 + (2\cdot 5 + 3\cdot 4)x + 3\cdot 5\\
&= 8x^2 + 22x + 15
\end{aligned}
$$

In order to reverse the multiplication, and to go back to what we started with by factoring, we need a way to break the middle term coefficient 22 up into the sum of the two original values $2\cdot 5$ and $3\cdot 4$. How can we do this?

We need to notice something in the pattern: $= \underbrace{2\cdot 4}_{A}x^2 + \underbrace{(2\cdot 5 + 3\cdot 4)}_{B}x + \underbrace{3\cdot 5}_{C}$

We will call the first coefficient A, the second B, and the third C. We notice that if we multiply $A\cdot C$ we get $2\cdot 4\cdot 3\cdot 5$ which can be rewritten as $(2\cdot 5)(3\cdot 4)$, which is just the product of the two parts that were added together to get B!

So to go from a trinomial of the form $Ax^2 + Bx + C$ back to the original four-term polynomial that resulted from the original multiplication, **we need find a pair of numbers that multiply to get AC and add to get B**.

This can be summed up by the following property:

$$ax^2 + bx + c = 0 \;\leftrightarrow\; ax^2 + b_1x + b_2x + c = 0 \quad where \begin{array}{l} *\, b_1, b_2 \text{ are whole numbers} \\ *\, b_1\cdot b_2 = ac \\ *\, b_1 + b_2 = b \end{array}$$

Let's try using this property to rewrite and factor trinomials (three terms):

<table>
<tr><th>Put into $Ax^2 + Bx + C$ form, if needed; then rewrite with four terms</th><th>Factor the four-term polynomial</th><th>Check your work by multiplying the factors</th></tr>
<tr><td>$10x^2 + 19x - 15$
$= 10x^2 + 19x + -15$

$A\cdot C = 10\cdot -15 = -150$
<table><tr><th colspan="2">All factors pairs of -150:</th></tr><tr><td>$-1, 150$</td><td>$1, -150$</td></tr><tr><td>$-2, 75$</td><td>$2, -75$</td></tr><tr><td>$-3, 50$</td><td>$3, -50$</td></tr><tr><td>$-5, 30$</td><td>$5, -30$</td></tr><tr><td>$-6, 25$</td><td>$6, -25$</td></tr><tr><td>$-10, 15$</td><td>$10, -15$</td></tr></table>Which add to make B, or positive 19?
$-6 + 25 = 19$

So we rewrite: $10x^2 + 19x + -15$
$\mathbf{= 10x^2 + -6x + 25x + -15}$</td><td>$10x^2 + -6x + 25x + -15$
$= (10x^2 + -6x) + (25x + -15)$
$= 2x(5x + -3) + 5(5x + -3)$
$= (2x + 5)(5x + -3)$ or
$= (2x + 5)(5x - 3)$</td><td>$(2x + 5)(5x - 3)$
$= (2x + 5)(5x + -3)$
$= 2x\cdot 5x + 2x\cdot -3 + 5\cdot 5x + 5\cdot -3$
$= 2\cdot 5\cdot x\cdot x + 2\cdot -3\cdot x + 25x + -15$
$= 10x^2 + -6x + 25x + -15$
$= 10x^2 + 19x + -15$ or
$\mathbf{= 10x^2 + 19x - 15}$
This is what we started with! ✓</td></tr>
<tr><td>$3p^2 - 7pq + 2q^2$
$= 3p^2 + -7pq + 2q^2$

$A\cdot C = 3\cdot 2 = 6$
<table><tr><th colspan="2">All factors pairs of -150:</th></tr><tr><td>$1, 6$</td><td>$-1, -6$</td></tr><tr><td>$2, 3$</td><td>$-2, -3$</td></tr></table>Which add to make B, or -7?
$-1 + -6 = -7$

So we rewrite: $3p^2 - 7pq + 2q^2$
$\mathbf{= 3p^2 + -1pq + -6pq + 2q^2}$</td><td>$3p^2 + -1pq + -6pq + 2q^2$
$= (3p^2 + -1pq) + (-6pq + 2q^2)$
$= p(3p + -q) + -2q(3p + -q)$

Notice: If we factor $2q$ out of the second set of parentheses above instead of $-2q$, we will end up with $-3p + q$ inside the second set of parentheses, and the two parentheses terms won't be like terms!

$= (p + -2q)(3p + -q)$ or
$= (p - 2q)(3p - q)$</td><td>$(p - 2q)(3p - q)$
$= (p + -2q)(3p + -q)$
$= p\cdot 3p + p\cdot -q + -2q\cdot 3p + -2q\cdot -q$
$= 3\cdot p\cdot p + -1pq + -2\cdot 3\cdot p\cdot q + -2q^2$
$= 3p^2 + -1pq + -6pq + -2q^2$
$= 3p^2 + -7pq + -2q^2$ or
$\mathbf{= 3p^2 - 7pq - 2q^2}$
This is what we started with! ✓</td></tr>
</table>

<table>
<tr>
<td>
$x^2 + 4x - 4$
$= x^2 + 4x + -4$

$A \cdot C = 1 \cdot -4 = -4$
<table>
<tr><th colspan="2">All factors pairs of -150:</th></tr>
<tr><td>$-1, 4$</td><td>$1, -4$</td></tr>
<tr><td>$-2, 2$</td><td></td></tr>
</table>
Which add to make B, or 4?
None of them!
So this trinomial is NOT FACTORABLE!
</td>
<td>NOT FACTORABLE</td>
<td>NOT FACTORABLE</td>
</tr>
<tr>
<td>
$4a^2 - 9$
$= 4a^2 + 0a + -9$

$A \cdot C = 4 \cdot -9 = -36$
<table>
<tr><th colspan="2">All factors pairs of -150:</th></tr>
<tr><td>$-1, 36$</td><td>$1, -36$</td></tr>
<tr><td>$-2, 18$</td><td>$2, -18$</td></tr>
<tr><td>$-3, 12$</td><td>$3, -12$</td></tr>
<tr><td>$-4, 9$</td><td>$5, -9$</td></tr>
<tr><td>$-6, 6$</td><td></td></tr>
</table>
Which add to make B, or 0?
$-6 + 6 = 0$

So we rewrite: $4a^2 + 0x + -9$
$= \mathbf{4a^2 + -6a + 6a + -9}$
</td>
<td>
$4a^2 + -6a + 6a + -9$
$= (4a^2 + -6a) + (6a + -9)$
$= 2a(2a + -3) + 3(2a + -3)$
$= (2a + 3)(2a + -3)$ or
$= (2a + 3)(2a - 3)$
</td>
<td>
$(2a + 3)(2a - 3)$
$= (2a + 3)(2a + -3)$
$= 2a \cdot 2a + 2a \cdot -3 + 3 \cdot 2a + 3 \cdot -3$
$= 2 \cdot 2 \cdot a \cdot a + 2 \cdot -3 \cdot a + 6a + -9$
$= 4a^2 + -6a + 6a + -9$
$= 4a^2 + 0 + -9$
$= 4a^2 + -9$ or
$= 4a^2 - 9$
</td>
</tr>
<tr>
<td>Now you try!
1) $2x^2 - 7x - 4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) $15y^2 - 36y + 12$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) $4a^2 + a - 6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) $2x^2 + xy - y^2$</td>
<td></td>
<td></td>
</tr>
</table>

5) $3x^2 + 6x - 5$		
6) $2a^2 - 7ab - 4b^2$		

Factoring Practice:

Expression to factor: **Is there GCF that can be factored out of each term? If so, do that here:**	**Are there four terms? If not, rewrite in $Ax^2 + Bx + C$ form if needed and then break into four terms here:**	**Is there further factoring (beyond factoring GCF out of each term)?** **If so, do that here:**	**Check your answer by multiplying all factors and check that you get the original expression:**
$2x^2 - 6x$ Yes $2x^2 - 6x = 2x^2 + -6x =$ $2x \cdot x + 2x \cdot -3$ $= 2x(x + -3)$ or $= 2x(x + 3)$	Yes	No	$2x(x - 3) = 2x(x + -3)$ $= 2x \cdot x + 2x \cdot -3$ $= 2x^2 + 2 \cdot -3 \cdot x$ $= 2x^2 + -6x$ or $2x^2 - 6x$ This is what we started with! ✓
$6xy - 4x + 3y - 2$ No	Yes	Yes $= 6xy + -4x + 3y + -2$ $= (6xy + -4x) + (3y + -2)$ $= 2x(3y + -2) + 1(3y + -2)$ $= (2x + 1)(3y + -2)$ or $= (2x + 1)(3y - 2)$	$(2x + 1)(3y - 2)$ $= (2x + 1)(3y + -2)$ $= 2x \cdot 3y + 2x \cdot -2 + 1 \cdot 3y + 1 \cdot -2$ $= 2 \cdot 3 \cdot x \cdot y + 2 \cdot -2 \cdot x + 3y + -2$ $= 6xy + -4x + 3y + -2$ or $= 6xy - 4x + 3y - 2$ This is what we started with! ✓
$30a^2b^2 - 10a^2b$ $+15ab^3 - 5ab^2$ Yes $= 30a^2b^2 + -10a^2b$ $+15ab^3 + -5ab^2$ $= 5ab \cdot 6ab + 5ab \cdot -2a$ $+5ab \cdot 3b^2 + 5ab \cdot -1b$ $= 5ab(6ab + -2a$ $+3b^2 + -1b)$	Yes	Yes $= 5ab([6ab + -2a]$ $+[3b^2 + -1b])$ $= 5ab(2a[3b \pm 1]$ $+b[3b + -1])$ $= 5ab([2a + b][3b + -1])$ $= 5ab(2a + b)(3b + -1)$ or $= 5ab(2a + b)(3b - 1)$	$5ab(2a + b)(3b - 1)$ $= 5ab(2a + b)(3b + -1)$ $= 5ab(2a \cdot 3b + 2a \cdot -1$ $+b \cdot 3b + b \cdot -1)$ $= 5ab(2 \cdot 3 \cdot a \cdot b + 2 \cdot -1 \cdot a$ $+3 \cdot b \cdot b + -1b)$ $= 5ab(6ab + -2a + 3b^2 + -1b)$ $= 5ab \cdot 6ab + 5ab \cdot -2a$ $+5ab \cdot 3b^2 + 5ab \cdot -1b$ $= 5 \cdot 6 \cdot a \cdot a \cdot b \cdot b + 5 \cdot -2 \cdot a \cdot a \cdot b$ $+5 \cdot 3 \cdot a \cdot b \cdot b^2 + 5 \cdot -1 \cdot a \cdot b \cdot b$ $= 30a^2b^2 + -10a^2b + 15a \cdot b \cdot b \cdot b$ $+ - 5ab^2$ $= 30a^2b^2 + -10a^2b + 15ab^3 + -5ab^2$ or $30a^2b^2 - 10a^2b + 15ab^3 - 5ab^2$ This is what we started with! ✓

<table>
<tr>
<td>$10x^2+19x-15$
No</td>
<td>No
$=10x^2+19x+-15$

$A\cdot C=10\cdot -15=-150$
<table><tr><th colspan="2">All factors pairs of -150:</th></tr><tr><td>$-1,150$</td><td>$1,-150$</td></tr><tr><td>$-2,75$</td><td>$2,-75$</td></tr><tr><td>$-3,50$</td><td>$3,-50$</td></tr><tr><td>$-5,30$</td><td>$5,-30$</td></tr><tr><td>$-6,25$</td><td>$6,-25$</td></tr><tr><td>$-10,15$</td><td>$10,-15$</td></tr></table>
Which add to make B, or positive 19?
$-6+25=19$

So we rewrite:
$10x^2+19x+-15$
$=\mathbf{10x^2+-6x+25x+-15}$</td>
<td>Yes
$10x^2+-6x+25x+-15$
$=(10x^2+-6x)$
$+(25x+-15)$
$=2x(5x+-3)+5(5x+-3)$
$=(2x+5)(5x+-3)$ or
$=(2x+5)(5x-3)$</td>
<td>$(2x+5)(5x-3)$
$=(2x+5)(5x+-3)$
$=2x\cdot 5x+2x\cdot -3+5\cdot 5x+5\cdot -3$
$=2\cdot 5\cdot x\cdot x+2\cdot -3\cdot x+25x+-15$
$=10x^2+-6x+25x+-15$
$=10x^2+19x+-15$ or
$=\mathbf{10x^2+19x-15}$
This is what we started with! ✓</td>
</tr>
<tr>
<td>$6x^4+-38x^3-28x^2$
Yes
$=6x^4+-38x^3+-28x^2$
$=2x^2(3x^2+-19x+-14)$</td>
<td>No
$2x^2(3x^2+-19x+-14)$

$A\cdot C=3\cdot -14=-42$
<table><tr><th colspan="2">All factors pairs of -42:</th></tr><tr><td>$-1,42$</td><td>$1,-42$</td></tr><tr><td>$-2,21$</td><td>$2,-21$</td></tr><tr><td>$-3,14$</td><td>$3,-14$</td></tr><tr><td>$-6,7$</td><td>$6,-7$</td></tr></table>
Which add to make B, or -19?
$2+-21=-19$

So we rewrite:
$2x^2(3x^2+-19x+-14)=$
$\mathbf{2x^2(3x^2+2x+-21x+-14)}$</td>
<td>$2x^2(3x^2+2x+-21x+-14)$
$=2x^2([3x^2+2x]$
$+[-21x+-14])$
$=2x^2(x[3x+2]$
$+-7[3x+2])$
$=2x^2([x+-7][3x+2])$
$=2x^2(x+-7)(3x+2)$ or
$=2x^2(x-7)(3x+2)$</td>
<td>$2x^2(x-7)(3x+2)$
$=2x^2(x+-7)(3x+2)$
$=2x^2(x\cdot 3x+x\cdot 2+-7\cdot 3x+-7\cdot 2)$
$=2x^2(3x^2+2x+-21x+-14)$
$=2x^2(3x^2+-19x+-14)$
$=2x^2\cdot 3x^2+2x^2\cdot -19x+2x^2\cdot -14$
$=2\cdot 3\cdot x\cdot x\cdot x\cdot x+2\cdot -19\cdot x\cdot x\cdot x$
$+2\cdot -14x^2$
$=6x^4+-38x^3+-28x^2$ or
$=6x^4+-38x^3-28x^2$
This is what we started with! ✓</td>
</tr>
<tr>
<td>x^2+4x-4
No</td>
<td>No
x^2+4x-4
$=x^2+4x+-4$

$A\cdot C=1\cdot -4=-4$
<table><tr><th colspan="2">All factors pairs of -150:</th></tr><tr><td>$-1,4$</td><td>$1,-4$</td></tr><tr><td>$-2,2$</td><td></td></tr></table>
Which add to make B, or 4?
None of them!
So this trinomial is **NOT FACTORABLE!**</td>
<td>**NOT FACTORABLE!**</td>
<td>**NOT FACTORABLE!**</td>
</tr>
<tr>
<td>$4a^2-9$
No</td>
<td>No
$4a^2-9$
$=4a^2+0a+-9$

$A\cdot C=4\cdot -9=-36$
<table><tr><th colspan="2">All factors pairs of -150:</th></tr><tr><td>$-1,36$</td><td>$1,-36$</td></tr><tr><td>$-2,18$</td><td>$2,-18$</td></tr><tr><td>$-3,12$</td><td>$3,-12$</td></tr><tr><td>$-4,9$</td><td>$5,-9$</td></tr><tr><td>$-6,6$</td><td></td></tr></table>
Which add to make B, or 0?
$-6+6=0$

So we rewrite: $4a^2+0x+-9$
$=\mathbf{4a^2+-6a+6a+-9}$</td>
<td>$4a^2+-6a+6a+-9$
$=(4a^2+-6a)+(6a+-9)$
$=2a(2a+-3)+3(2a+-3)$
$=(2a+3)(2a+-3)$ or
$=(2a+3)(2a-3)$</td>
<td>$(2a+3)(2a-3)$
$=(2a+3)(2a+-3)$
$=2a\cdot 2a+2a\cdot -3+3\cdot 2a+3\cdot -3$
$=2\cdot 2\cdot a\cdot a+2\cdot -3\cdot a+6a+-9$
$=4a^2+-6a+6a+-9$
$=4a^2+0+-9$
$=4a^2+-9$ or
$=4a^2-9$
This is what we started with! ✓</td>
</tr>
</table>

1) $6x^4 + 18x^2$			
2) $10pq + 25p + 4q + 10$			
3) $2x^2 + 7x - 4$			
4) $y^2 - 36$			
5) $6x^3 - 4x^2 - 3x + 2$			
6) $6x^2 - 7xy + 3y^2$			
7) $12x^3 - 75x$			
8) $6ac - 2ad - 3bc + bd$			
9) $18b^4 - 51b^3 + 36b^2$			
10) $4a^3 - 20a^2 + 4a$			
11) $4x^2yz - 8x^2y + 2x^2z - 4x^2$			

Solving quadratic equations:

Sometimes we can solve quadratic equations the same way we solved linear equations. For example, let's solve this:
Solve for x: $3x^2 = 27$

Steps to solve	Explanation
$3x^2 = 27$	
$\frac{3x^2}{3} = \frac{27}{3}$	We want to move the 3 to get x by itself; it is currently multiplying, so we need to divide by 3. We divide both sides completely by 3.
$1x^2 = 9$	
$x^2 = 9$	
$x = -3$ or $x = 3$	What can be squared to get 9? We have to remember that both positive and negative numbers when squared produced a positive number, so we notice that both 3 and -3 can be squared to get 9, and in fact these are the only two numbers that can be squared to get 9.

However, most quadratic equations have more than just an x^2 term in them, and therefore can't be solved this way because we run into the following problem. If we try to get x by itself on one side in this quadratic equation: $x^2 + 4x - 4 = 0$, we can't do it, **because the x^2 term and the $4x$ term are NOT like terms, so there is no way to combine them.**

So to solve quadratic equations, we have to use a different approach. This requires us to notice an important pattern:

If we multiply two things together and get zero as the result (called the product), at least one of the two original things that we multiplied has to be zero. It is not possible to multiple two non-zero numbers and get zero as the answer. We can write this property using math symbols like this: $a \cdot b = 0 \rightarrow a = 0 \ or \ b = 0 \ (or \ both)$.

So, to solve quadratic equations, we just need to rewrite them as a product of several factors that are equal to zero. We can do this by:

1) Moving everything over to one side, to get zero on the other side, and then;
2) Factoring everything on the non-zero side;
3) Then we can simply set each factor equal to zero and solve.

So, for example, to solve $x^2 = 5x - 6$ for x, we simply need to follow these steps:

1) Find an equivalent equation with everything over to one side and zero on the other side:

$x^2 = 5x - 6$
$x^2 = 5x + -6$
$(x^2) + -5x = (5x + -6) + -5x$
$x^2 + -5x = 5x + -6 + -5x$
$x^2 + -5x = 5x + -5x + -6$
$x^2 + -5x = 0 + -6$
$x^2 + -5x = -6$
$(x^2 + -5x) + 6 = (-6) + 6$
$x^2 + -5x + 6 = 0$

1) Factor the left side:
$1 \cdot 6 = 6$

Factors of 6	
1,6	$-1, -6$
2,3	$-2, -3$

$-2 + -3 = -5$
$(x - 2)(x - 3) = 0$

2) Set each factor equal to zero:

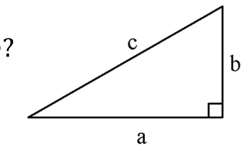
$x - 2 = 0$	or	$x - 3 = 0$
$x + -2 = 0$	or	$x + -3 = 0$
$(x + -2) + 2 = 0 + 2$	or	$(x + -3) + 3 = 0 + 3$
$x + -2 + 2 = 2$	or	$x + -3 + 3 = 3$
$x + 0 = 2$	or	$x + 0 = 3$
$x = 2$	or	$x = 3$

We notice that when solving a quadratic equation, there should always be two solutions (except in cases where the two factors are identical—e.g. When we have $(x - 3)(x - 3) = 0$), we get two instances of $x - 3 = 0$ and so we get $x = 3$ twice, leaving just the solution $x = 3$.

Applications: The Pythagorean Theorem

This theorem gives a formula for how the sides of a <u>right</u> triangle relate to one another: $\boldsymbol{a^2 + b^2 = c^2}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the short sides of a right triangle, and $\boldsymbol{c}$ is the long side, or hypotenuse, of the right triangle. If we know two of these three values, we can write out a quadratic equation and solve for the third value. For example:

Consider the following right triangle, where $a = 4$ and $c = 10$. What is the value of b?



To solve this we write: $(4)^2 + b^2 = (10)^2$

$16 + b^2 = 100$

$(16 + b^2) + -16 = (100) + -16$

$16 + b^2 + -16 = 84$

$b^2 + 16 + -16 = 84$

$b^2 = 84$

$\sqrt{b^2} = \pm\sqrt{84}$

$b = \pm\sqrt{2 \cdot 2 \cdot 3 \cdot 7}$

$b = \pm\sqrt{2^2 \cdot 3 \cdot 7}$

$b = \pm\sqrt{2^2} \cdot \sqrt{3 \cdot 7}$

$b = \pm 2\sqrt{21}$

$b = 2\sqrt{21}$ (answer must be positive—it is a distance)

Now you try! Solve the following quadratic equations:

Can this equation be solved by taking the square root, or do you need to factor in order to solve?	**Solve the equation for the given variable.**	**Check your answer(s) by substituting them into the original equation.**
1) $50x^2 = 450$		
2) One side of a right triangle is 6 in and the hypotenuse is 10 in. Find the length of the remaining side.		
3) $(3x - 2)(x + 5) = 0$		
4) $7a - 3 = 6a^2$		
5) The diagonal of your computer screen is 20 in. One side is 16 in. How long is the other side?		
6) $9b^2 - 16 = 0$		
7) $3x^2 + 12 = 0$		
8) Two sides of a right triangle are 5 ft and 10 ft. How long is the hypotenuse?		

9) $3z^2 = 48$		
10) $2x^2 + 7x = 4$		
11) One side of a right triangle has a side of 6 ft and a hypotenuse of 9 ft. How long is the other side?		
12) $2y(y - 1)(2y + 3) = 0$		
13) $2x^2 + 7x = 7 - 5x$		
14) $5a^2 - 15 = 0$		

Translating between words and math: Common words and the math symbols they represent

Expression part	Words indicating this operation	Order
Addition	plus, sum, total, altogether, and, add(ed), increase by, more than	Order doesn't matter for addition.
Subtraction	minus, subtract from, difference between/of, less than, decreased/reduced by, take away from	**ASK:** *Which quantity are you starting with, and which quantity are you taking away from that starting value?* • a minus $b \leftrightarrow a - b$ • a decreased/reduced by $b \leftrightarrow a - b$ Whatever comes after "by" is what is being taken away, even if other words have a different order. • difference between/of a and $b \leftrightarrow a - b$ • subtract/take away a from $b \leftrightarrow b - a$ Whatever comes after "from" is the starting value, even if other words have a different order. • a less than $b \leftrightarrow b - a$
Multiplication	multiply, product, times, of, twice/doubled (times 2)	Order doesn't matter for multiplication.
Division	divide(d) by, quotient/ratio of, per, go(es) into	**ASK:** *Which quantity are you starting with, and which quantity is dividing up the original value?* • a divided by $b \leftrightarrow a \div b$ or $\frac{a}{b}$ Whatever comes after "by" is what is doing the dividing, even if other words have a different order. • a per $b \leftrightarrow a \div b$ or $\frac{a}{b}$ • Quotient/ratio of a and $b \leftrightarrow a \div b$ or $\frac{a}{b}$ a goes into $b \leftrightarrow b \div a$ or $\frac{b}{a}$ Whatever comes after "into" is the starting value, even if other words have a different order.
Exponents	squared, cubed, to the power of	
Equals sign	is/are/was/were, yields, makes	
Variable	a number	Can pick any letter—it's up to you!

Subtraction/Division Practice:

Sentence	Which is the original quantity?	Write using math symbols
1. The difference between four and a number	four	$4 - n$
2. The quotient of six and a number		
3. A number decreased by two		
4. Six goes into a number		
5. Eight miles per two gallons		
6. Five reduced by a number		
7. Take five away from a number		
8. From six, take away a number		
9. Divide three into a number		
10. Divide a number by three		
11. A number minus seven		
12. Take three away from a number		
13. From three, take away a number		
14. The difference between six and two		

Multiple operations at once and parentheses:

Sentence	Does the first operation apply to the <u>second quantity</u> mentioned, or the <u>result</u> of the second operation?	Write using math symbols
Three times five, minus four	Second quantity (5)	$(3 \cdot 5) - 4$ or $3 \cdot 5 - 4$
Three times the difference between five and four	Result (difference)	$3 \cdot (5 - 4)$
The product of nine more than a number and two	Result (nine more than a number) *Note that "the product of nine" doesn't make sense on its own—a product needs two things to be multiplied!*	$(9 + n) \cdot 2$
The product of two and nine, plus a number	Second quantity (9)	$(2 \cdot 9) + n$ or $2 \cdot 9 + n$
The difference of two and a number, times seven	Second quantity (a number)	$(2 - x) \cdot 7$
The difference of a number times seven and two	Result (a number times seven) *Note that "the difference of a number" doesn't make sense on its own—a difference needs two things: the original value and the amount being taken away!*	$(x \cdot 7) - 2$ or $x \cdot 7 - 2$
Eight less than three times a number	Result (three times a number)	$3n - 8$
Eight less than three, times a number	Second quantity (three)	$(3 - 8)n$
15. The ratio of nine and a number, minus five		
16. The ratio of nine less than a number and five		
17. Six less than the total of a number and five		
18. Six less than a number, plus five		
19. The sum of four times a number and two		
20. The sum of four and two, times a number		
21. Twice the difference of five and a number		
22. The difference of twice five and a number		
23. Five less than a number divided by five		
24. Five less than a number, divided by five		
25. A number subtracted from three, times four		
26. A number subtracted from three times four		
27. Six times the difference of a number and two		
28. The difference of six times a number and two		
29. Three times the square of a number		
30. Subtract two from the cube of a number		